Exercise 8: Stress and strain

04.12.2023 - 08.12.2023

Question 1 . . . .

Reference: Barber, Elasticity, Springer (2010), p. 32

Plastic deformation during a manufacturing process generates a state of stress in the large body $z > 0$. If the stresses are functions of z only and the surface $z = 0$ is not loaded, show that the stress components σ_{yz}, σ_{zx}, σ_{zz} must be zero everywhere!

Question 2 . . . .

Metal or semiconductor crystals may contain defects in their lattice structure called “dislocations”. These are very important for understanding plastic deformation. A so-called “screw dislocation”, sketched in the figure, is created by the following displacement

$$\mathbf{u}(x,y,z) = \begin{bmatrix} 0 \\ 0 \\ \frac{b}{2\pi}\arctan\left(\frac{y}{x}\right) \end{bmatrix}.$$

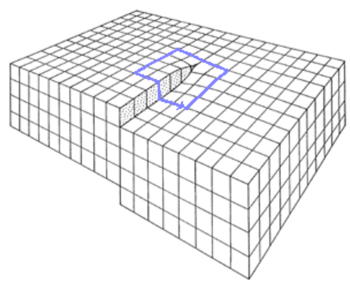

Figure 1: screw dislocation from: https://www.tf.uni -kiel.de/matwis/amat/def_en/kap_5/backbone/r5_2_2.html

Calculate the associated strain tensor ε and the stress tensor σ (using Hooke’s law)! Is the body in a state of plane strain or plane stress? Do you notice something peculiar near the center of the dislocation at $x = y = 0$?

Question 3 . . . .

We now consider a state of plane strain. The governing equations are

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y}, \quad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) \quad \text{(definition of strain)},$$

$$\sigma_{xx} = 2\mu\varepsilon_{xx} + \lambda\left(\varepsilon_{xx} + \varepsilon_{yy}\right), \quad \sigma_{yy} = 2\mu\varepsilon_{yy} + \lambda\left(\varepsilon_{xx} + \varepsilon_{yy}\right), \quad \sigma_{xy} = 2\mu\varepsilon_{xy} \quad \text{(Hooke's law)},$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + F_x = 0, \quad \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} + F_y = 0, \quad \text{(equilibrium)}.$$

These are eight governing equations. However, we can combine them in such a way that we end up with only two equations in terms of the displacement components u_x and u_y. This form is convenient for problems where displacement components are prescribed over the entire boundary of the body. Find these two equations!

Question 4 . . . .

We want to demonstrate for the two-dimensional case that Hooke’s law with isotropic elastic constants is indeed isotropic. Consider a 2D stress tensor σ and the corresponding strain ε,

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix}.$$

Next, consider the matrix for rotation by an arbitrary angle α

$$R = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix}.$$

The most straightforward way to demonstrate isotropy would be to rotate the elastic stiffness tensor. However, this is a fourth-order tensor and rotating it is cumbersome. Here, we take a different approach. In order to demonstrate isotropy

1. express σ in terms of the components of ε,
2. rotate σ to find the representation σ' of this state of stress in the new coordinate system,
3. replace the components of ε in σ' by the components of the strain tensor ε' in the rotated coordinate system.

You should see that the constants of proportionality between stress and strain — the elastic constants — are the same in the new and the old coordinate system!