Exercise 6: Stress invariants

27.11.2023 - 01.12.2023

Question 1

Analyse the plane stress

$$\underline{\sigma} = \begin{pmatrix} 3 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 2 \end{pmatrix}$$

(a) Find the rotation angle $\phi_{\sigma,\max}$ at which the diagonal stress entries are maximal.

(b) Which values take the principal or main stresses σ_1 and σ_2? You can use the formulas derived in the lecture.

(c) As well find the rotation angle $\phi_{\tau,\max}$ at which the shear stress is maximal and compute the value for the maximal shear stress $\tau_{\max}$.

(d) In the lecture it was shown that not only the principal stresses can characterize a stress state but also the stress invariants. Compute the stress invariants I_1 and I_2.

(e) The dimension of a stress as well as the dimension of the principal stresses is force per area. What are the dimensions of the two stress invariants I_1 and I_2.

Question 2

The following stress tensor characterises a special stress state

$$\underline{\sigma} = \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -\sqrt{2} \end{pmatrix}$$

(a) Compute the angle $\phi_{\sigma,\max}$ at which the normal stresses takes its maximal value.

(b) Use the general rotation matrix and the computed angle $\phi_{\sigma,\max}$ to rotate the stress state in the coordinate system of maximal normal stress. What are the values for the principal stresses?

(c) What is the special name for the stress state found in (b)?

(d) In the lecture we have derived two rotation angles to rotate the stress from maximal diagonal elements into the coordinate system where the shear stress is maximal. Take one of the two angles and rotate the stress computed in (b) by the rotation matrix to find the stress state with maximal shear stress.

Question 3

Now we have a more general three dimensional stress state given by

$$\underline{\sigma} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

(a) Compute the three principal stresses which are the eigenvalues of the stress tensor.

(b) What are the values of the three invariants I_1, I_2 and I_3 of the given stress state?

(c) Compute the hydrostatic stress σ_h.

(d) Compute the deviatoric stress s_{ij} which is also called stress deviator.

(e) Which values take the invariants J_1, J_2 and J_3 of the stress deviator.

(f) What is the value of the von Mises stress?

(g) What is special about J_2 and why is the von Mises stress derived from J_2?