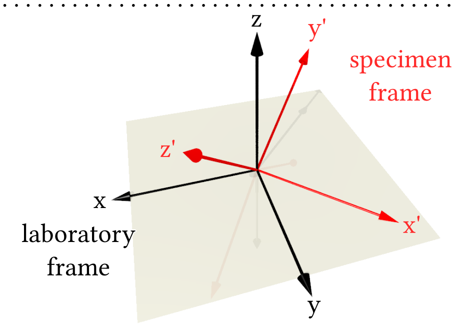
# Exercise 5: Stress tensor and recap

**20.11.2023 - 24.11.2023**

**Question 1** . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

In this exercise, we will practice tensor rotation. Consider the two coordinate systems in the figure on the right. The red coordinate system ("specimen frame") has been rotated. The basis vectors of this system with respect to the laboratory frame are




$$\mathbf{x}' = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\\0\end{bmatrix}, \qquad \mathbf{y}' = \frac{1}{\sqrt{6}}\begin{bmatrix}1\\1\\-2\end{bmatrix}, \qquad \mathbf{z}' = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}.$$

Suppose we are given the representation of a stress tensor in the *specimen frame*,

$$\boldsymbol{\sigma}' = \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & \sigma_{zz}\end{bmatrix}.$$

This stress tensor would be created by a force that acts on the plane whose normal is $\mathbf{z}'$, along the $\mathbf{z}'$-direction. We want the representation $\boldsymbol{\sigma}$ of this stress tensor in the *laboratory frame*.

(a) Find the rotation matrix $\mathbf{R}$ which, given the representation of a vector in the laboratory frame, yields the representation in the specimen frame upon matrix-vector multiplication!

(b) Verify that the determinant of $\mathbf{R}$ is equal to 1!

(c) Perfom tensor rotation to obtain $\boldsymbol{\sigma}$!

(d) Calculate the von Mises stress for $\boldsymbol{\sigma}'$ and $\boldsymbol{\sigma}$!

**Question 2** . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

Compute the following expressions in Einstein notation and afterwards rewrite the expression in the dyadic notation to recap your tensor algebra skills. For some expressions you have also learned names, find them.

The tensors are given as follows: $v_1 = 2, v_2 = 1$; $A_{11} = 1, A_{12} = 2, A_{21} = 3, A_{22} = 4$; $B_x = 3x^2 + y, B_y = 3z - xy, B_z = xyz$; $C_{xx} = 2xy, C_{xy} = xy, C_{yx} = x^2 - 4y, C_{yy} = y^2$

(a) $A_{i1}$

(b) $A_{ij}A_{ji}$

(c) $A_{ji}v_jA_{kk}$

(d) $B_{i,j}$

(e) $C_{ij,k}$

(f) $C_{ij,j}$

**Question 3** . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

In figure 1 you see a beam which is supported by a roller and a pinned support. On the left side a constant line load of $\tilde{q}(x) = q_0$ is applied in the region $0 \le x \le a$. In between the two supports at $3/2a$ a point force $F$ acts in z-direction.

(a) Describe the point force by a single line load q(x). Use therefore the delta distribution.

(b) Now you can find the shear force $\frac{\mathrm{d}Q(x)}{\mathrm{d}x} = -q(x)$ and the moment $\frac{\mathrm{d}^2M(x)}{\mathrm{d}x^2} = -q(x)$ by integrating the two differential equations. You have to compute the integrals for the left $(0 \le x \le a)$ and right $(a \le x \le 2a)$ sector independently.

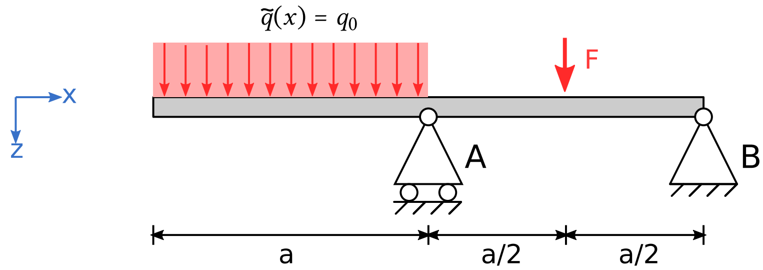



Figure 1: Beam with two supports.

(c) Why do you have to compute the integrals for each sector?

(d) You should have now two unknown constants in the solutions for $Q(x)$ and $M(x)$ in each sector. By using the boundary conditions you can find three equations and an other equation follows from the continuity of the momentum along the bar, especially in $x = a$.

(e) Compute the reaction forces at the two supports from the previous derived solution.

(f) Proof your results for the reaction forces at support A and B by computing them from the static equilibrium of the forces.

(g) Compute the internal moments at $x = a$ and $x = 2a$. Note that although there is a roller support in $x = a$ the moment can be unequal to zero. This is not to the case for $x = 2a$. Can you understand the difference in the support A and B (besides that one is a roller and the other one a pinned support)?