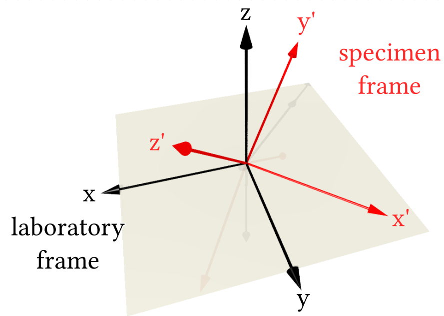
Exercise 5: Stress tensor and recap

20.11.2023 - 24.11.2023

Question 1 ....

In this exercise, we will practice tensor rotation. Consider the two coordinate systems in the figure on the right. The red coordinate system ("specimen frame") has been rotated. The basis vectors of this system with respect to the laboratory frame are



$$\mathbf{x}' = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\\0\end{bmatrix}, \qquad \mathbf{y}' = \frac{1}{\sqrt{6}}\begin{bmatrix}1\\1\\-2\end{bmatrix}, \qquad \mathbf{z}' = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}.$$

Suppose we are given the representation of a stress tensor in the *specimen frame*,

$$\boldsymbol{\sigma}' = \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & \sigma_{zz}\end{bmatrix}.$$

This stress tensor would be created by a force that acts on the plane whose normal is $\mathbf{z}'$, along the $\mathbf{z}'$-direction. We want the representation $\boldsymbol{\sigma}$ of this stress tensor in the *laboratory frame*.

(a) Find the rotation matrix $\mathbf{R}$ which, given the representation of a vector in the laboratory frame, yields the representation in the specimen frame upon matrix-vector multiplication!

(b) Verify that the determinant of $\mathbf{R}$ is equal to 1!

(c) Perfom tensor rotation to obtain $\boldsymbol{\sigma}$!

(d) Calculate the VON MISES stress for $\boldsymbol{\sigma}'$ and $\boldsymbol{\sigma}$!

Solution:

(a)
The task is to find the rotation matrix $\mathbf{R}$ which rotates a vector from the laboratory frame (L) into the specimen frame (S), given the axes $(\mathbf{x}', \mathbf{y}', \mathbf{z}')$ of the specimen frame coordinate system. We make this clear by calling this rotation $\mathbf{R}_{L\to S}$. To find the rotation $\mathbf{R}_{L\to S}$ we will first derive the rotation matrix $\mathbf{R}_{cL\to cS}$ for the axes of the coordinate systems. So $\mathbf{R}_{cL\to cS}$ rotates the coordinate system of the laboratory frame into the coordinate system of the specimen frame, i.e.

$$\mathbf{R}_{cL\to cS}\cdot\mathbf{x} = \mathbf{x}' \quad , \qquad \mathbf{R}_{cL\to cS}\cdot\mathbf{y} = \mathbf{y}' \quad , \qquad \mathbf{R}_{cL\to cS}\cdot\mathbf{z} = \mathbf{z}'$$

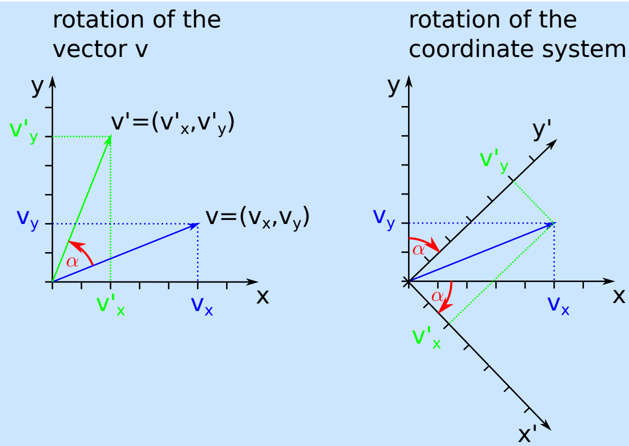
We already know the axes of both systems and find the rotation matrix as follows,

$$\mathbf{R}_{cL\to cS}\cdot\mathbf{x} = \begin{pmatrix} r_{c,xx} & r_{c,xy} & r_{c,xz}\\ r_{c,yx} & r_{c,yy} & r_{c,yz}\\ r_{c,zx} & r_{c,zy} & r_{c,zz}\end{pmatrix}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}r_{c,xx}\\ r_{c,yx}\\ r_{c,zx}\end{pmatrix} \stackrel{!}{=} \mathbf{x}' = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\\0\end{bmatrix}$$

By analogous computations for $\mathbf{y}$ and $\mathbf{z}$ we find

$$\mathbf{R}_{cL\to cS} = (\mathbf{x}, \mathbf{y}, \mathbf{z}) = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}}\end{bmatrix}$$

However this is the rotation matrix for the coordinate systems. To find the rotation matrix for a vector remember the following.



As shown in the above sketch it does not matter whether you rotate the vector or the coordinate system, you will find the same coefficients for v'_x and v'_y. However, the rotation indicated by the red arrow is in the opposite direction. Thus the rotation for a vector is the inverse of the rotation for the coordinate system. Using also the unitarity $(R^{-1} = R^T)$ of rotation matrices we find

$$\mathbf{R}_{L\to S} = \mathbf{R}^{-1}_{cL\to cS} = \mathbf{R}^{T}_{cL\to cS} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

(b)
Recall that

$$\begin{aligned} \det(\mathbf{R}) &= \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix} \\ &= (R_{11}R_{22}R_{33} + R_{12}R_{23}R_{31} + R_{13}R_{21}R_{32}) - (R_{31}R_{22}R_{13} + R_{32}R_{23}R_{11} + R_{33}R_{21}R_{12}) \,. \end{aligned}$$

For $\mathbf{R}_{L\to S}$ you thus find $\det(\mathbf{R}_{L\to S}) = 1$.

(c)
To keep it shorter we use here $\mathbf{R}$ for the rotation $\mathbf{R}_{L\to S}$. In dyadic notation the rotation of the second order tensor σ' can be found as explained in the lecture. We want to rotate the stress tensor from the specimen frame into the laboratory frame. By analysing the rotation behaviour of vectors we can figure out how one has to rotate a second order tensor. From (a) we know

$$\begin{aligned} & \mathbf{R}\cdot\mathbf{v} = \mathbf{v}' \\ \Leftrightarrow\quad & \mathbf{R}^T\cdot\mathbf{R}\cdot\mathbf{v} = \mathbf{R}^T\cdot\mathbf{v}' \\ \Leftrightarrow\quad & \mathbf{v} = \mathbf{R}^T\mathbf{v}' \end{aligned}$$

For a second order tensor $\mathbf{A}'$ and a vectors $\mathbf{v}'$ and $\mathbf{b}'$ in the specimen frame we find

$$\begin{aligned} & \mathbf{A}'\cdot\mathbf{v}' = \mathbf{b}' \\ \Leftrightarrow\quad & \mathbf{R}^T\cdot(\mathbf{A}'\cdot\mathbf{v}') = \mathbf{R}^T\cdot\mathbf{b}' \\ \Leftrightarrow\quad & \mathbf{R}^T\cdot(\mathbf{A}'\cdot\underbrace{\mathbf{R}\cdot\mathbf{R}^T}_{=\mathbb{1}}\cdot\mathbf{v}') = \underbrace{\mathbf{R}^T\cdot\mathbf{b}'}_{=\mathbf{b}} \\ \Leftrightarrow\quad & \underbrace{\mathbf{R}^T\cdot\mathbf{A}'\cdot\mathbf{R}}_{=\mathbf{A}}\cdot\underbrace{\mathbf{R}^T\cdot\mathbf{v}'}_{=\mathbf{v}} = \mathbf{b} \end{aligned}$$

So we have found:

$$\sigma = \mathbf{R}^T \cdot \sigma' \cdot \mathbf{R}$$

In Einstein sum notation one can write:

$$\sigma_{mn} = R_{im}\sigma'_{ij}R_{jn}.$$

Note that only $\sigma_{33} = \sigma_{zz}$ is nonzero. Therefore, we need to consider only one term for each mn, namely

$$\sigma_{mn} = R_{3m}R_{3n}\sigma'_{33} = R_{3m}R_{3n}\sigma_{zz}.$$

Furthermore, keep in mind that the rotated stress tensor must be symmetric. Hence

$$\begin{aligned}
\sigma_{11} &= R_{31}^2\sigma_{zz} = \frac{1}{3}\sigma_{zz}, \\
\sigma_{22} &= R_{32}^2\sigma_{zz} = \frac{1}{3}\sigma_{zz}, \\
\sigma_{33} &= R_{33}^2\sigma_{zz} = \frac{1}{3}\sigma_{zz}, \\
\sigma_{13} &= \sigma_{31} = R_{31}R_{33}\sigma_{zz} = \frac{1}{3}\sigma_{zz}, \\
\sigma_{23} &= \sigma_{32} = R_{32}R_{33}\sigma_{zz} = \frac{1}{3}\sigma_{zz}, \\
\sigma_{12} &= \sigma_{12} = R_{31}R_{32}\sigma_{zz} = \frac{1}{3}\sigma_{zz}.
\end{aligned}$$

An in matrix form

$$\boldsymbol{\sigma} = \frac{1}{3}\sigma_{zz}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

(d)
Recall the definition of the von Mises stress in terms of the components of the stress tensor σ_{ij}, or in terms of the deviatoric stress $s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$

$$\sigma_{vM} = \sqrt{\frac{3}{2}s_{ij}s_{ij}} = \sqrt{\frac{1}{2}\left[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2\right] + 3\left(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2\right)}.$$

For $\boldsymbol{\sigma}'$ we have

$$\sigma'_{vM} = \sqrt{\frac{1}{2}\left[\sigma_{33}^2 + \sigma_{33}^2\right]} = \sigma_{zz}.$$

And for $\boldsymbol{\sigma}$

$$\sigma_{vM} = \sqrt{3\left(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2\right)} = \sigma_{zz}.$$

As expected, the two take the same value. The von Mises stress is an invariant of the stress tensor.

Question 2 . . . .

Compute the following expressions in Einstein notation and afterwards rewrite the expression in the dyadic notation to recap your tensor algebra skills. For some expressions you have also learned names, find them.

The tensors are given as follows: $v_1 = 2, v_2 = 1;\ A_{11} = 1, A_{12} = 2, A_{21} = 3, A_{22} = 4;\ B_x = 3x^2 + y, B_y = 3z - xy, B_z = xyz;\ C_{xx} = 2xy, C_{xy} = xy, C_{yx} = x^2 - 4y, C_{yy} = y^2$

(a) A_{i1}

(b) $A_{ij}A_{ji}$

(c) $A_{ji}v_jA_{kk}$

(d) $B_{i,j}$

(e) $C_{ij,k}$

(f) $C_{ij,j}$

Solution: First of all it might be useful to write the tensors down in their matrix and vector representation:

$$\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad \vec{B}(x,y,z) = \begin{pmatrix} 3x^2 + y \\ 3z - xy \\ xyz \end{pmatrix} \qquad C(x,y) = \begin{pmatrix} 2xy & xy \\ x^2 - 4y & y^2 \end{pmatrix}$$

(a) $A_{i1} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ which is the first column of the matrix A

(b) $A_{ij}A_{ji} = \sum_{i,j} A_{ij}A_{ji} = (A_{11}A_{11} + A_{12}A_{21}) + (A_{21}A_{12} + A_{22}A_{22}) = A_{11}^2 + 2A_{12}A_{21} + A_{22}^2 = 29$
In dyadic notation one would write: $A_{ij}A_{ji} = \mathrm{tr}(A \cdot A)$

(c) $A_{ji}v_jA_{kk} = \sum_j A_{ji}v_j \sum_k A_{kk} = \begin{pmatrix} A_{11}v_1 + A_{21}v_2 \\ A_{12}v_1 + A_{22}v_2 \end{pmatrix} (A_{11} + A_{22}) = \begin{pmatrix} 25 \\ 40 \end{pmatrix}$
In dyadic notation one would write: $A_{ji}v_jA_{kk} = A^T \cdot \vec{v}\,\mathrm{tr}(A)$

(d) $B_{i,j} = \partial_j B_i = \begin{pmatrix} \partial_x B_x & \partial_y B_x & \partial_z B_x \\ \partial_x B_y & \partial_y B_y & \partial_z B_y \\ \partial_x B_z & \partial_y B_z & \partial_z B_z \end{pmatrix} = \begin{pmatrix} 6x & 1 & 0 \\ -y & -x & 3 \\ yz & xz & xy \end{pmatrix}$
In dyadic notation one would write: $B_{i,j} = \partial_j B_i = \vec{\nabla}\vec{B} = \mathrm{grad}(\vec{B})$, which is the gradient of $\vec{B}$.

(e) In dyadic notation one would write: $C_{ij,k} = \partial_k C_{ij} = \vec{\nabla} C = \mathrm{grad}(C)$, which is the gradient of C.

$$C_{ij,k} = \partial_k C_{ij} = \begin{pmatrix} \partial_x C_{ij} & \partial_y C_{ij} \end{pmatrix} = \left(\begin{pmatrix} \partial_x C_{11} & \partial_x C_{12} \\ \partial_x C_{21} & \partial_x C_{22} \end{pmatrix} \quad \begin{pmatrix} \partial_y C_{11} & \partial_y C_{12} \\ \partial_y C_{21} & \partial_y C_{22} \end{pmatrix} \right)$$
$$= \left(\begin{pmatrix} 2y & y \\ 2x & 0 \end{pmatrix} \quad \begin{pmatrix} 2x & x \\ -4 & 2y \end{pmatrix} \right)$$

(f) $C_{ij,j} = \sum_j \partial_j C_{ij} = \begin{pmatrix} \partial_1 C_{11} + \partial_2 C_{12} \\ \partial_1 C_{21} + \partial_2 C_{22} \end{pmatrix} = \begin{pmatrix} 2y + x \\ 2x + 2y \end{pmatrix}$
In dyadic notation one would write: $C_{ij,j} = \sum_j \partial_j C_{ij} = C \cdot \vec{\nabla} = \mathrm{div}(C)$, which is the divergence of C.

Question 3 . . . .
In figure 1 you see a beam which is supported by a roller and a pinned support. On the left side a constant line load of $\tilde{q}(x) = q_0$ is applied in the region $0 \leq x \leq a$. In between the two supports at $3/2a$ a point force F acts in z-direction.

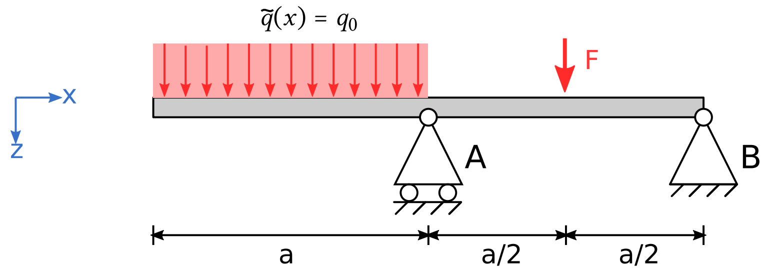


Figure 1: Beam with two supports.

(a) Describe the point force by a single line load q(x). Use therefore the delta distribution.

(b) Now you can find the shear force $\frac{\mathrm{d}Q(x)}{\mathrm{d}x} = -q(x)$ and the moment $\frac{\mathrm{d}^2M(x)}{\mathrm{d}x^2} = -q(x)$ by integrating the two differential equations. You have to compute the integrals for the left $(0 \leq x \leq a)$ and right $(a \leq x \leq 2a)$ sector independently.

(c) Why do you have to compute the integrals for each sector?

(d) You should have now two unknown constants in the solutions for $Q(x)$ and $M(x)$ in each sector. By using the boundary conditions you can find three equations and an other equation follows from the continuity of the momentum along the bar, especially in $x = a$.

(e) Compute the reaction forces at the two supports from the previous derived solution.

(f) Proof your results for the reaction forces at support A and B by computing them from the static equilibrium of the forces.

(g) Compute the internal moments at $x = a$ and $x = 2a$. Note that although there is a roller support in $x = a$ the moment can be unequal to zero. This is not to the case for $x = 2a$. Can you understand the difference in the support A and B (besides that one is a roller and the other one a pinned support)?

Solution:

(a) Before starting to solve the task it is useful to check if it is solvable. Therefore check if the setup is statically determined $(n = 1, r = 1 + 2, v = 0 \Rightarrow (3n - (r + v)) = 0 \Rightarrow$ statically determined!). Now you can start. $q(x) = F\delta(x - \frac{3}{2}a)$

(b) By integrating the differential equations we find the formulas also derived in the lecture.

$$Q(x) = \int -q(x)\mathrm{d}x \quad , \qquad M(x) = \int Q(x)\mathrm{d}x$$

For the left sector $(0 \leq x \leq a)$, indicated by an upper index "l", we find:

$$Q^l(x) = \int -\tilde{q}(x)\mathrm{d}x = \int -q_0\mathrm{d}x = -q_0x + C_1$$
$$M^l(x) = \int Q^l(x)\mathrm{d}x = -\frac{1}{2}q_0x^2 + C_1x + C_2$$

and for the right sector ($a \le x \le 2a$), indicated by an upper index "r", we find:

$$Q^r(x) = \int -q(x)\mathrm{d}x = \int -F\,\delta\left(x - \frac{3}{2}a\right)\mathrm{d}x = -F\,H\left(x-\frac{3}{2}a\right) + C_1'$$
$$M^r(x) = \int Q^r(x)\mathrm{d}x = -F\left(x-\frac{3}{2}a\right)\,H\left(x-\frac{3}{2}a\right) + C_1'x + C_2'$$

(c) You have to separate the integral in two sectors because the support in $x = a$ generates a discontinuity in the shear force. This discontinuity is of up to now unknown height and you need additional degrees of freedom to fix this "jump" in the shear force.

(d) To fix the four constants C_1, C_2, C_1' and C_2' we use the three boundary conditions and the intermediate condition of a continuous moment at $x = a$, namely

$$\begin{aligned}
Q^l(0) &\overset{!}{=} 0 \quad &\Rightarrow\quad & C_1 = 0\\
M^l(0) &\overset{!}{=} 0 \quad &\Rightarrow\quad & C_2 = 0\\
M^r(2a) &\overset{!}{=} 0 \quad &\Rightarrow\quad & -F\frac{a}{2} + C_1'2a + C_2' = 0\\
M^l(a) &\overset{!}{=} M^r(a) \quad &\Rightarrow\quad & -\frac{1}{2}q_0a^2 = C_1'a + C_2'
\end{aligned}$$

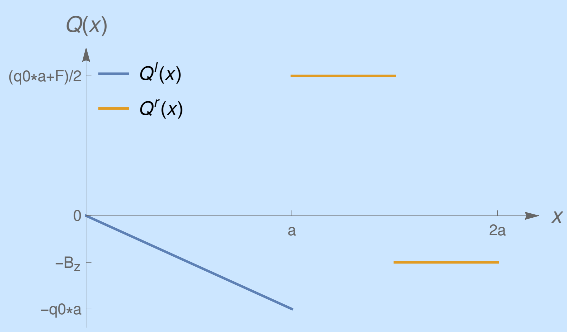
By solving the last to equations we find

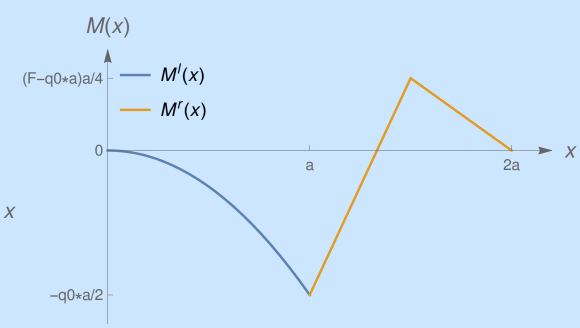
$$C_1' = \frac{1}{2}q_0a + \frac{1}{2}F \quad , \qquad C_2' = -\frac{1}{2}Fa - q_0a^2$$

which gives

$$\begin{aligned}
Q^l(x) &= -q_0x\\
M^l(x) &= -\frac{1}{2}q_0x^2\\
Q^r(x) &= -F\,H\left(x-\frac{3}{2}a\right) + \frac{1}{2}q_0a + \frac{1}{2}F\\
M^r(x) &= -F\left(x-\frac{3}{2}a\right)\,H\left(x-\frac{3}{2}a\right) + \left(\frac{1}{2}q_0a + \frac{1}{2}F\right)x - \frac{1}{2}Fa - q_0a^2
\end{aligned}$$

Now it is also possible to plot the moment and the shear force along the beam which is useful to understand the system better. The plot is done for $q_0a = F/2$.





(e) At Support A:

$$Q^l(a) = -q_0a \quad , \qquad Q^r(a) = \frac{1}{2}q_0a + \frac{1}{2}F$$

Because at $x = a$ there is a support which creates a reaction force in z-direction the shear force is not continuous at this point. The difference in the shear force from left to right is exactly the reaction force created by support A. Thus we find

$$A_z = Q^r(a) - Q^l(a) = \frac{3}{2}q_0 a + \frac{1}{2}F$$

The discontinuity in the shear force created by the reaction force of support A is the reason why you have to separate the bar in a left and a right sector.
At Support B:

$$Q^r(2a) = -\frac{1}{2}F + \frac{1}{2}q_0 a = -B_z$$

We find only a solution by $Q^r(2a)$, the shear force which has to be hold by support B, thus $B_z = -Q^r(2a)$. The force in x-direction is obvious zero and thus was not handled in the computations, $B_x = 0$.

Comment:
Sometimes it might not be easy to chose the right signs in the equations above. For example it is not obvious why to compute $A_z = Q^r(a) - Q^l(a)$ and not $A_z = Q^l(a) - Q^r(a)$. To get the right signs it is necessary to first know about the sign convention we chose in the lecture, namely the shear force vector $Q(x)$ is pointing upwards. For this exercise that means $Q(x)$ is pointing in negative z-direction. Now you have two options to find the right sign/direction of the reaction force from support A.
Option 1: Have a look at the shear force plot from (d). At $x = a$ the shear force has a positive change (keep in mind the direction of x, so the shear force goes from negative to positive and not the other way round) so the reaction force at A, which is responsible for the positive step in $Q(x)$, has to point in the direction of $Q(x)$. This means you can take the absolute value of the step, $|Q^r(a) - Q^l(a)|$, and let it point into positive direction of $Q(x)$. Because $Q(x)$ and A_z are pointing in the same direction ($Q(x)$ by definition upwards, A_z as sketched in (f)) we find $A_z = +|Q^r(a) - Q^l(a)| = +|Q^l(a) - Q^r(a)|$.
Option 2: is a more mathematical answer. We start from the definition of the shear force

$$\frac{dQ(x)}{dx} = -q(x)$$

and describe the reaction force A_z as unknown point force in the point $x = a$. The reaction force A_z indicated in part (f) is pointing in negative z-direction which leads to the expression

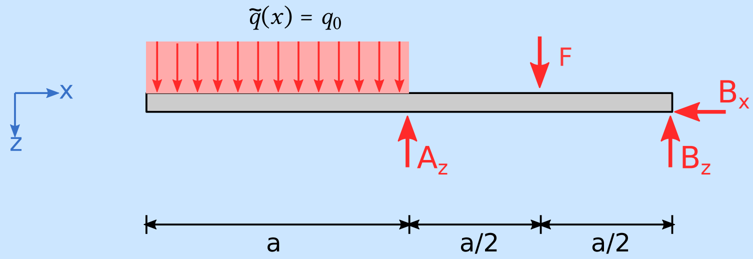
$$q(x) = \underbrace{-}_{\text{negative z-direction}|}\ \underbrace{A_z}_{\text{absolute value}|}\ \underbrace{\delta(x-a)}_{\text{point force at } x=a} = -A_z\delta(x-a)$$

We are only interested in the point $x = a$, let $\epsilon > 0$ be a small value. Now we integrate the definition of the shear force in the point $x = a$ to find the reaction A_z

$$\begin{aligned}
& \lim_{\epsilon\to 0}\int_{a-\epsilon}^{a+\epsilon}\frac{dQ(x)}{dx}\mathrm{d}x = \lim_{\epsilon\to 0}\int_{a-\epsilon}^{a+\epsilon} -q(x)\mathrm{d}x \\
\Leftrightarrow\quad & \lim_{\epsilon\to 0}\int_{a-\epsilon}^{a+\epsilon}\frac{dQ(x)}{dx}\mathrm{d}x = \lim_{\epsilon\to 0}\int_{a-\epsilon}^{a+\epsilon} -(-A_z\delta(x-a))\mathrm{d}x \\
\Leftrightarrow\quad & \lim_{\epsilon\to 0}\left[Q(x)\right]_{a-\epsilon}^{a+\epsilon} = \lim_{\epsilon\to 0}\int_{a-\epsilon}^{a+\epsilon} -(-A_z\delta(x-a))\mathrm{d}x \\
\Leftrightarrow\quad & \lim_{\epsilon\to 0}\left[Q(x)\right]_{a-\epsilon}^{a+\epsilon} = A_z \\
\Leftrightarrow\quad & \lim_{\epsilon\to 0}\left(Q(a+\epsilon) - Q(a-\epsilon)\right) = A_z \\
\Leftrightarrow\quad & \left(Q^r(a) - Q^l(a)\right) = A_z
\end{aligned}$$

The same arguments can be done for the reaction force B_z.

(f) Draw the cut free beam with all forces



$$\rightarrow \qquad -B_x = 0 \quad \Leftrightarrow \quad B_x = 0$$

$$\uparrow \qquad \int_0^a q_0 \mathrm{d}x + F - A_z - B_z = 0 \quad \Leftrightarrow \quad aq_0 + F - A_z - B_z = 0$$

$$\text{A} \qquad \int_0^a q_0(a-x)\mathrm{d}x - F\frac{a}{2} + B_z a = 0 \quad \Leftrightarrow \quad \frac{1}{2}q_0 a^2 - F\frac{a}{2} + B_z a = 0$$

By solving the equations we find:

$$A_z = \frac{3}{2}q_0 a + \frac{1}{2}F \quad , \qquad B_z = \frac{1}{2}F - \frac{1}{2}q_0 a$$

Which is equivalent to our findings from the integration method.

(g) For the internal moments at $x = a$ and $x = 2a$ we find

$$M^l(a) = M^r(a) = -\frac{1}{2}q_0 a \neq 0$$
$$M^r(2a) = 0$$

Even though there is a roller support A at $x = a$ the internal moment is unequal to zero. This is the case because the roller support is not at a free end of the bar. The support only can generate a force in z-direction and can not supply a force in x-direction neither a moment. It is important to recognize that the missing ability to supply a moment is not equal to the condition of having a zero internal moment. The internal moments can also be balanced by the moments in the bar. This is the case at any other cut of the bar where the moment from the left side balances the moment from the right side.
In point B however we have a zero internal moment. The important difference to A is that at B the bar has a free end. Thus, there is only one internal moment from the left side of the bar. The only other moment acting in point B is the generated moment of support B which is by definition zero. Hence there is only the internal moment which thus has to be zero to fulfil the balance of moments.