Exercise 4: Divergence and static equilibrium
13.11.2023 - 17.11.2023

Question 1 . . . .

Reference: Chou, Pagano, *Elasticity: Tensor, Dyadic, and Engineering Approaches*, Dover Publications, pages 32–33.

(a) Is the following stress distribution possible for a body in equilibrium? A, B, and C are constants. Body forces are zero.

$$\begin{aligned}
\sigma_{xx} &= -Axy \\
\tau_{xy} &= \frac{A}{2}\left(B^2 - y^2\right) + Cz \\
\tau_{xz} &= -Cy \\
\sigma_{yy} &= \sigma_{zz} = \tau_{yz} = 0
\end{aligned}$$

(b) Check whether equilibrium exists for the following stress distribution. Body forces are zero.

$$\begin{aligned}
\sigma_{xx} &= 3x^2 + 4xy - 8y^2 \\
\sigma_{yy} &= 2x^2 + xy + 3y^2 \\
\tau_{xy} &= -\frac{1}{2}x^2 - 6xy - 2y^2 \\
\sigma_{zz} &= \sigma_{zx} = \tau_{yz} = 0
\end{aligned}$$

Question 2 . . . .

You might have some problems in computing and getting a feeling of a divergence. In the lecture you have learned about the divergence theorem, also called the Gauss theorem, which can help you in understanding the divergence of a vector field.

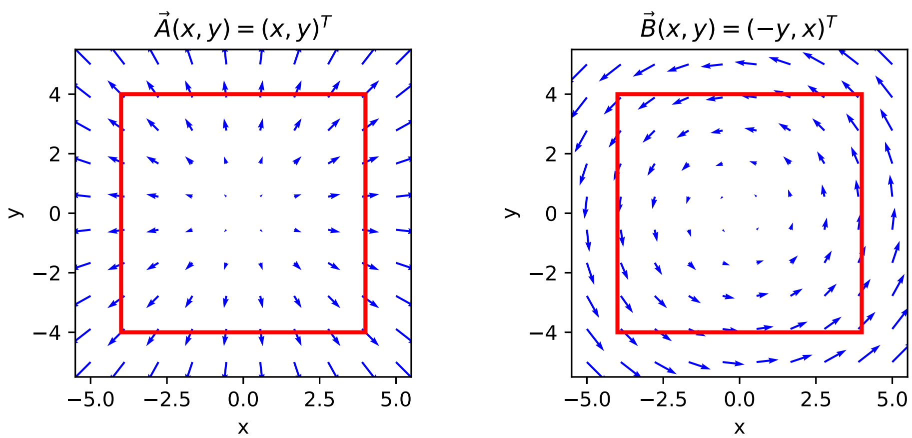

Figure 1: Two different vector fields $\vec{A}$ and $\vec{B}$, indicated by arrows in blue. In red the surface $\partial\Omega$ enclosing the area Ω used for the divergence theorem.

(a) Let us start in computing the divergence of the two vector fields sketch in figure 1. The first vector field $\vec{A}(x,y) = \begin{pmatrix} x \\ y \end{pmatrix}$ could be the velocity of water coming out of the tip from a cone and running down at the surface. The second one, $B(x,y) = \begin{pmatrix} -y \\ x \end{pmatrix}$, could be the velocity of particles fixed on a rotating plate.

(b) Now you can have a look at the divergence theorem

$$\int_{\Omega} \vec{\nabla} \cdot \vec{F} \mathrm{d}\Omega = \int_{\partial\Omega} \vec{n} \cdot \vec{F} \mathrm{d}\partial\Omega$$

for a vector field $\vec{F}$ and the area Ω enclosed by the surface $\partial\Omega$ with surface normal vector $\vec{n}$ pointing outwards. Compute the left **and** the right hand side of the divergence theorem for the vector fields $\vec{A}$ and $\vec{B}$ form part (a). The area Ω is given by the red box in the figure. To compute the right hand side of the divergence theorem it is useful to subdivide the integral into four parts, the four sides of the square.

(c) Now try to interpret your findings from (a) and (b). You can therefore imagine squares of arbitrary size at different positions in the two plots of the vector fields. How does the vector field behave/change inside the square areas? Are there ”more” vectors pointing out of the area than coming in? Is there a relation to the divergence computed in (a)? Can you describe the quantities computed by the left and the right side of the divergence theorem in (b) pictorially in the figures?